



Calhoun: The NPS Institutional Archive
DSpace Repository

Theses and Dissertations

1. Thesis and Dissertation Collection, all items

1963

An Analysis of the Cost and Requirements of the Fleet Ballistic Missile Submarine Personnel Subsystem

Baldwin, Edwin M.

Monterey, California: U.S. Naval Postgraduate School

<http://hdl.handle.net/10945/11603>

This publication is a work of the U.S. Government as defined in Title 17, United States Code, Section 101. Copyright protection is not available for this work in the United States.

Downloaded from NPS Archive: Calhoun



Calhoun is the Naval Postgraduate School's public access digital repository for research materials and institutional publications created by the NPS community. Calhoun is named for Professor of Mathematics Guy K. Calhoun, NPS's first appointed -- and published -- scholarly author.

Dudley Knox Library / Naval Postgraduate School
411 Dyer Road / 1 University Circle
Monterey, California USA 93943

<http://www.nps.edu/library>

NPS ARCHIVE
1963
BALDWIN, E.

AN ANALYSIS OF THE COST AND REQUIREMENTS
OF THE FLEET BALLISTIC MISSILE SUBMARINE
PERSONNEL SUBSYSTEM
EDWIN M. BALDWIN

AT-480917

LIBRARY

U.S. NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA

[Add to Shopping Cart](#)[Save for Later](#)

Technical Reports Collection

Citation Format: Full Citation (1F)

Accession Number :

AD0480913

Citation Status:

Active

Citation Classification:

Unclassified

Field(s) & Group(s):

050300 - ECONOMICS AND COST ANALYSIS

050900 - PERSONNEL MANAGEMENT AND LABOR RELATIONS

131001 - SUBMARINE ENGINEERING

Corporate Author:

NAVAL POSTGRADUATE SCHOOL MONTEREY CA

Unclassified Title:AN ANALYSIS OF THE COST AND REQUIREMENTS OF THE FLEET BALLISTIC MISSILE
SUBMARINE PERSONNEL SUBSYSTEM.**Title Classification:**

Unclassified

Descriptive Note:

Master's thesis,

Personal Author(s):

Baldwin, Edwin M.

Report Date:

01 Jan 1963

Media Count:

37 Page(s)

Cost:

\$7.00

Report Classification:

Unclassified

Descriptors:*BALLISTIC MISSILE SUBMARINES), (*SUBMARINE PERSONNEL, (*COST EFFECTIVENESS,
SUBMARINE PERSONNEL), NAVAL PERSONNEL, NUCLEAR POWERED SHIPS, TIME
STUDIES, OPERATIONS RESEARCH, DEPLOYMENT, MATHEMATICAL ANALYSIS, COSTS,
SUBMARINES, EQUATIONS, MATHEMATICAL MODELS**Abstract:**

The Polaris Submarine Weapons System is presently employing two crews per submarine, working in shifts in order to maximize the time on station for each submarine. The cost of this personnel policy as compared to the cost of using other alternatives is of current interest. The purpose of this thesis is to present a technique for making this comparison. The actual data are not presented here, but hypothetical examples are given. (Author)

Abstract Classification:

Unclassified

Annotation:

LIBRARY

U.S. NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA

An analysis of the cost and requirements of the Fleet Ballistic Missile Submarine Personnel Subsystem - Thesis.

Distribution Limitation(s):

01 - APPROVED FOR PUBLIC RELEASE

Source Code:

251450

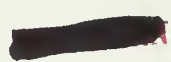
Document Location:

DTIC

Change Authority:

ST-A USNPS LTR 24 NOV 71

AN ANALYSIS
OF THE COST AND REQUIREMENTS
OF THE
FLEET BALLISTIC MISSILE SUBMARINE
PERSONNEL SUBSYSTEM



* * * * *

Edwin M. Baldwin

RELEASABLE TO NPS INTERNATIONAL AFFILIATES

(Ref.: Memo, T702 NC4(0142) dtd 21 Mar 83

This document is subject to special export controls and each transmittal to foreign government or foreign nationals may be made only with prior approval of the U.S. Naval Postgraduate School (Code 035).



AN ANALYSIS
OF THE COST AND REQUIREMENTS
OF THE
FLEET BALLISTIC MISSILE SUBMARINE
PERSONNEL SUBSYSTEM

by

Edwin M. Baldwin

Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE

United States Naval Postgraduate School
Monterey, California

1 9 6 3

AN ANALYSIS
OF THE COST AND REQUIREMENTS
OF THE
FLEET BALLISTIC MISSILE SUBMARINE
PERSONNEL SUBSYSTEM

by

Edwin M. Baldwin

This work is accepted as fulfilling
the thesis requirements for the degree of
MASTER OF SCIENCE
from the

United States Naval Postgraduate School

ABSTRACT

The polaris Submarine Weapons System is presently employing two crews per submarine, working in shifts in order to maximize the time on station for each submarine. The cost of this personnel policy as compared to the cost of using other alternatives is of current interest. The purpose of this thesis is to present a technique for making this comparison. The actual data is not presented here, but hypothetical examples are given.

TABLE OF CONTENTS

	Page
Introduction	1
Chapter I. The Operational Cycles	3
Chapter II. The Analytical Model	13
1. The Number of Boats Per Station	13
2. The Long Range Personnel Requirements	14
3. The Cost Equations	15
4. The Choice and Effect of Upkeep Base Location	18
Chapter III. Analytical Model	20
1. The Alternatives	20
2. Examples	26
Chapter IV Conclusions and Recommendations	29
1. Conclusions	29
2. Recommendations	29
Appendix	31
List of symbols	36

LIST OF FIGURES AND TABLES

	Page
Figures	
1. The BOAT PATROL CYCLE	3
2. The Relationship Between the CREW PATROL CYCLE and the BOAT PATROL CYCLE	4
3. LONG RANGE OPERATIONAL CYCLE	6
4. SSBN PERSONNEL CAREER CYCLE	8
5. The Relationship Between the LONG RANGE OPERATIONAL CYCLE and the SSBN PERSONNEL CAREER CYCLE	9
6. The SSBN Personnel System	10
Tables	
1. Equations and Format for the Analysis	23
2. Suggested Measures of Effectiveness	25

PREFACE

The Introduction describes the problem studied in this paper and the nature of the results obtained. The major factors and essential elements of the problem are also briefly discussed.

Chapter I provides a description of the operation of the Polaris submarines in terms of two sets of operational cycles, one set describing the boat operation and one set describing the personnel system. These cycles consist of patrol, maintenance, leave, training, and yard work phases.

Chapter II of this paper presents the analytical model for making the comparison between alternative personnel policies. The method of determining the number of submarines and personnel required to keep one boat on station continuously for a period of years is developed first. Then the cost equations giving the boat costs and personnel expenses per boat on station are given. Finally, the relationship between the crew equations and the boat equations is shown in order to develop the criterion for selecting upkeep base location.

In Chapter III, the alternative personnel policies are discussed, and the procedures for using the analysis of this paper are described. Four examples are provided for illustration.

Chapter IV presents the Conclusion and Recommendations

The author wishes to acknowledge the help of his advisor, Professor T. E. Oberbeck, who kept him from going astray at several crucial points, and Associate Professor J. H. Gandelman, who provided a great deal of advice and assistance.

INTRODUCTION

The Polaris Submarine Weapon System operation presently employs two crews per submarine in order to maximize the amount of time each submarine can be on station. This thesis presents a method of comparing alternatives to the present dual crew policy by comparing the numbers of personnel and submarines required and the costs associated with each policy.

The factors which bear most heavily on this problem are:

- The number of crews per boat
- The sea duty-shore duty rotation plan
- The personnel attrition rate
- The cost of training
- The annual cost of the pay and administration of the personnel
- The initial cost of the submarines and equipment
- The annual cost of operation and maintenance
- The number of submarines required to keep one on station.

It is assumed that the average expenses associated with each crew and the cost of a submarine can be determined from budgetary data and personnel statistics. The cost of the boat with its equipment and the expenses associated with the personnel will be considered separately.

A submarine is considered to be capable of spending a fraction of its "life" on patrol while the remaining part is spent in maintenance, either in the shipyard or tender upkeep base. The fraction of time a boat can spend on station determines the number of boats required to patrol one station continuously. Likewise, the personnel are capable of spending only a fraction of their careers on station; thus, it will take a certain number of personnel, more than one crew, to keep a boat on station.

The personnel are subject to attrition. Because of attrition, new personnel must be trained each year to keep the crews up to full

complement. It is training this input of personnel that makes up a significant portion of the overall crew costs. Thus the rate of loss of personnel from the Polaris system is an important factor. The determination of this factor is complicated by the unpredictability of the personnel reaction to each alternative personnel policy. It will be possible to estimate, at least qualitatively, the effect on the attrition rate that various alternatives may bring about. With this problem in mind, the results are expressed as functions of the attrition rate.

For each alternative personnel policy, the results can be used to determine such quantities as:

The number of stations which can be patrolled with a given number of submarines

The number of personnel which must be trained over a period of years

The personnel training costs and other personnel expenses per station patrolled

The boat, missile, and equipment costs per station patrolled

The conclusions that are reached are in the form of equations because the actual figures to be compared by this model have not yet been collected. Comparisons using hypothetical values are given in Chapter III.

CHAPTER I

THE OPERATIONAL CYCLES

The Polaris Weapon System operation can be characterized by several operational cycles. In this section, each of these cycles is described in terms of a certain sequence of events. From the cycles, it will be observed that the personnel and submarines spend only certain fractions of their "careers" on station, ready to fire. These fractions are important; therefore, they are given definitions and symbols to represent them.

The fundamental cycle is the BOAT PATROL CYCLE. It consists of an UPKEEP period and a PATROL period and is illustrated in Figure 1. The UPKEEP period will usually take place at an advanced tender upkeep base, but may take place at the home port of the submarine or elsewhere within the Continental United States. The PATROL will consist of a transit to station, the on-station period, and the transit back to the base. The fraction of the BOAT PATROL CYCLE spent on patrol is an important variable; therefore, we define this fraction as:

$$(1) \quad t_b = \frac{(\text{Length of Patrol})}{(\text{Length of Boat Patrol Cycle})}$$

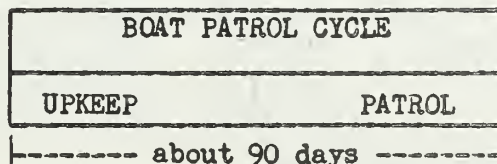


FIGURE 1. The BOAT PATROL CYCLE

The CREW PATROL CYCLE is similar to the BOAT PATROL CYCLE except that it must be modified to accommodate the use of multiple crews. The CREW PATROL CYCLE consists of an UPKEEP period, a PATROL period, and in

the case of multiple crews, a HOME PORT LEAVE AND TRAINING period as shown in Figure 2. The CREW PATROL CYCLES and the BOAT PATROL CYCLES must be fitted together to provide a smooth working plan in which each crew in turn takes the boat through a complete BOAT PATROL CYCLE.

For one crew per boat:

	BOAT PATROL CYCLE	
BOAT	UPKEEP	PATROL
CREW	CREW PATROL CYCLE	

For Two crews per boat:

	BOAT PATROL CYCLE		BOAT PATROL CYCLE	
BOAT	UPKEEP	PATROL	UPKEEP	PATROL
CREW I	UPKEEP	PATROL	HOME PORT LEAVE & TRAINING	
CREW II	HOME PORT LEAVE & TRAINING		UPKEEP	PATROL
	CREW PATROL CYCLE			

For three crews per boat:

	BOAT PATROL CYCLE		BOAT PATROL CYCLE		BOAT PATROL CYCLE	
BOAT	UPKEEP	PATROL	UPKEEP	PATROL	UPKEEP	PATROL
CREW I	UPKEEP	PATROL	HOME PORT LEAVE & TRAINING			
CREW II	HOME PORT LEAVE & TRAINING		UPKEEP	PATROL	HOME PORT LEAVE & TRAINING	
CREW III	HOME PORT LEAVE & TRAINING				UPKEEP	PATROL
	CREW PATROL CYCLE					

FIGURE 2. The relationships between the BOAT PATROL CYCLES and the CREW PATROL CYCLES

In the present situation and in many of the alternative personnel policies, one or more crews will be used to operate each deployed boat; therefore, it will be convenient to define

- (2) a = the number of deployed crews per
deployed boat.¹

Notice that

$$a \times (\text{Length of the BOAT PATROL CYCLE}) = \\ (\text{Length of the CREW PATROL CYCLE}).$$

We have previously defined a variable (t_b) which indicates the fraction of time which the boats spend on patrol. For the crews, we are interested in the fraction of time they are away from their home port. There are two important cases. The UPKEEP periods may all take place in the advanced base, or they may all take place in the home port. In a third case, which is developed in detail in Section 4 of Chapter II, there may be a plan to schedule some upkeep in home port and some in the advanced base.

- (3) t_c = the fraction of the CREW PATROL CYCLE that
the crew is away from home.

¹The terms "DEPLOYMENT" and "DEPLOYED" are used in this paper in a special sense. Boats on "Deployment" and Crews on "Deployment" are those boats and crews which are actively engaged in the business of keeping Polaris missiles on station, which in this paper will include the upkeep periods and crew leave and training periods between patrols.

1. The first part of the document is a letter from the President of the United States to the Congress.

2. The second part is a report on the state of the Union.

3. The third part is a report on the state of the Treasury.

4. The fourth part is a report on the state of the Navy.

5. The fifth part is a report on the state of the War Department.

6. The sixth part is a report on the state of the Department of the Interior.

7. The seventh part is a report on the state of the Department of Justice.

8. The eighth part is a report on the state of the Department of Education.

9. The ninth part is a report on the state of the Department of Agriculture.

10. The tenth part is a report on the state of the Department of Commerce.

11. The eleventh part is a report on the state of the Department of the Army.

12. The twelfth part is a report on the state of the Department of the Navy.

13. The thirteenth part is a report on the state of the Department of the War.

14. The fourteenth part is a report on the state of the Department of the Interior.

15. The fifteenth part is a report on the state of the Department of Justice.

16. The sixteenth part is a report on the state of the Department of Education.

17. The seventeenth part is a report on the state of the Department of Agriculture.

18. The eighteenth part is a report on the state of the Department of Commerce.

19. The nineteenth part is a report on the state of the Department of the Army.

20. The twentieth part is a report on the state of the Department of the Navy.

21. The twenty-first part is a report on the state of the Department of the War.

22. The twenty-second part is a report on the state of the Department of the Interior.

23. The twenty-third part is a report on the state of the Department of Justice.

24. The twenty-fourth part is a report on the state of the Department of Education.

25. The twenty-fifth part is a report on the state of the Department of Agriculture.

In the case when the UPKEEP is in the advanced base

$$t_c = \frac{\text{Length of Patrol and Upkeep}}{\text{Length of CREW PATROL CYCLE}}$$

and

$$t_c = \frac{\text{Length of BOAT PATROL CYCLE}}{\text{Length of CREW PATROL CYCLE}} - \frac{1}{a}$$

For the case when the UPKEEP takes place in the home port,

$$t_c = \frac{\text{Length of Patrol}}{\text{Length of CREW PATROL CYCLE}}$$

$$= \frac{\text{Length of Patrol}}{a \cdot (\text{Length of BOAT PATROL CYCLE})} = \frac{t_b}{a}$$

Therefore, we write

$$(3a) \quad t_c = \begin{cases} 1/a & \text{if all UPKEEP takes place in the} \\ & \text{advanced base} \\ t_b/a & \text{if all UPKEEP takes place in home} \\ & \text{port.} \end{cases}$$

In considering the extent to which the boat is deployed over a long period of time, it is convenient to define the LONG RANGE OPERATIONAL CYCLE, which shall consist of a BOAT DEPLOYMENT phase and a YARD OVERHAUL. The BOAT DEPLOYMENT phase shall consist of a number of BOAT PATROL CYCLES as shown in Figure 3. The YARD OVERHAUL phase shall consist of the entire period between deployments which includes the preparation for the yard work, the yard period itself, and the post yard shakedown and training period.

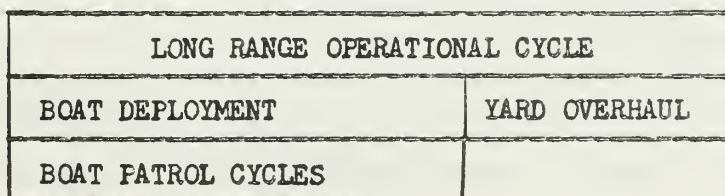


FIGURE 3. The LONG RANGE OPERATIONAL CYCLE of the FBM Submarine.

Consideration of Figure 3 leads us to define

- (4) T_D = fraction of the LONG RANGE OPERATIONAL CYCLE spent in the DEPLOYMENT phase.

Then,

$$\begin{aligned} T_D &= \frac{\text{Length of BOAT DEPLOYMENT}}{\text{Length of LONG RANGE OPERATIONAL CYCLE}} \\ &= \frac{P \times \text{Length of BOAT PATROL CYCLE}}{\text{Length of LONG RANGE OPERATIONAL CYCLE}} \end{aligned}$$

To determine the number of submarines needed to keep one submarine on station at all times, we must determine the amount of time needed for upkeep and yard maintenance during the submarine's lifetime. The larger the portion of the submarine's life spent in the shipyard and in maintenance, the more submarines we need to keep one on station at all times.

For the crews we have a similar situation. In addition to assignment to a deployed SSBN, each submariner expects and requires a certain amount of his career to be spent in other types of Navy activity. Here, too, we shall need more than one man for each billet to fill continuously the crews of the deployed submarine. We are also faced with the fact that crewmen become more senior in years and experience and must be shifted from billet to billet to take advantage of their capabilities to handle more responsible positions as time goes on. At some point in time, every man trained and put into the FBM program will either reach retirement age or leave at some earlier time.

The assignment to a deployed crew will not be permanent, therefore it is necessary to consider an overall career rotation plan. Let us define an SSBN PERSONNEL CAREER CYCLE which consists of the SSBN ASSIGNMENT phase and the ALTERNATE ASSIGNMENT phase. Figure 4 illustrates

this cycle and figure 5 illustrates all of the cycles.

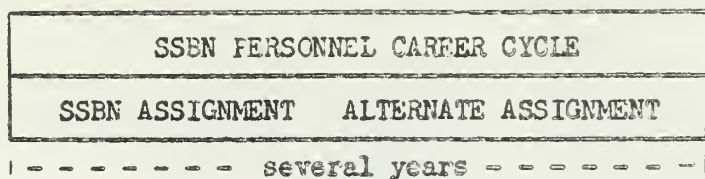


FIGURE 4. THE SSBN PERSONNEL CAREER CYCLE

Every man trained for duty in FBM submarines who remains qualified will be in the PERSONNEL CAREER CYCLE. During the SSBN ASSIGNMENT phase each FBM submariner will be assigned to one of the crews of the deployed submarines. During the ALTERNATE ASSIGNMENT phase, the FBM submariner will be assigned to other billets in the FBM program or to other locations in the Navy. For convenience, we shall consider those personnel assigned to SSBN located in the yard as being in the ALTERNATE ASSIGNMENT phase. It is only when a man retires, or leaves the SSBN program permanently, that he will be counted as "lost by attrition."

The more detailed model of the CAREER CYCLE shows the deployed crews and the non deployed crews. The deployed crews are the ones which take turns taking the submarine through the BOAT PATROL CYCLE and thus they rotate through the CREW PATROL CYCLE. The non deployed or "pseudo crews", as they will be called here, are the ones to which the personnel in the ALTERNATE ASSIGNMENT phase are conceptually assigned. These pseudo crews do not rotate, but the personnel assigned to them do rotate to the deployed crews and back to the pseudo crews. Figure 6 displays this model graphically, with the training input and attrition also shown.

To assist in accounting for the numbers of those crews, we define for any given personnel policy,

$$(5) \quad a' = \begin{array}{l} \text{the number of pseudo crews per} \\ \text{deployed boat,} \end{array}$$

and

LONG RANGE OPERATIONAL CYCLE							
BOAT	BOAT DEPLOYMENT					YARD	
	BOAT PATROL CYCLE		BOAT PATROL CYCLE		BOAT PATROL CYCLE		
	UPKEEP	PATROL	UPKEEP	PATROL	UPKEEP		PATROL
CREW	UPKEEP	PATROL	LEAVE AND TRAINING		UPKEEP	PATROL	
	CREW PATROL CYCLE				CREW PATROL CYCLE		
	SSBN ASSIGNMENT					ALTERNATE ASSIGNMENT	
	SSBN PERSONNEL CAREER CYCLE						

FIGURE 5. RELATION BETWEEN LONG RANGE OPERATIONAL CYCLE
AND SSBN PERSONNEL CAREER CYCLE

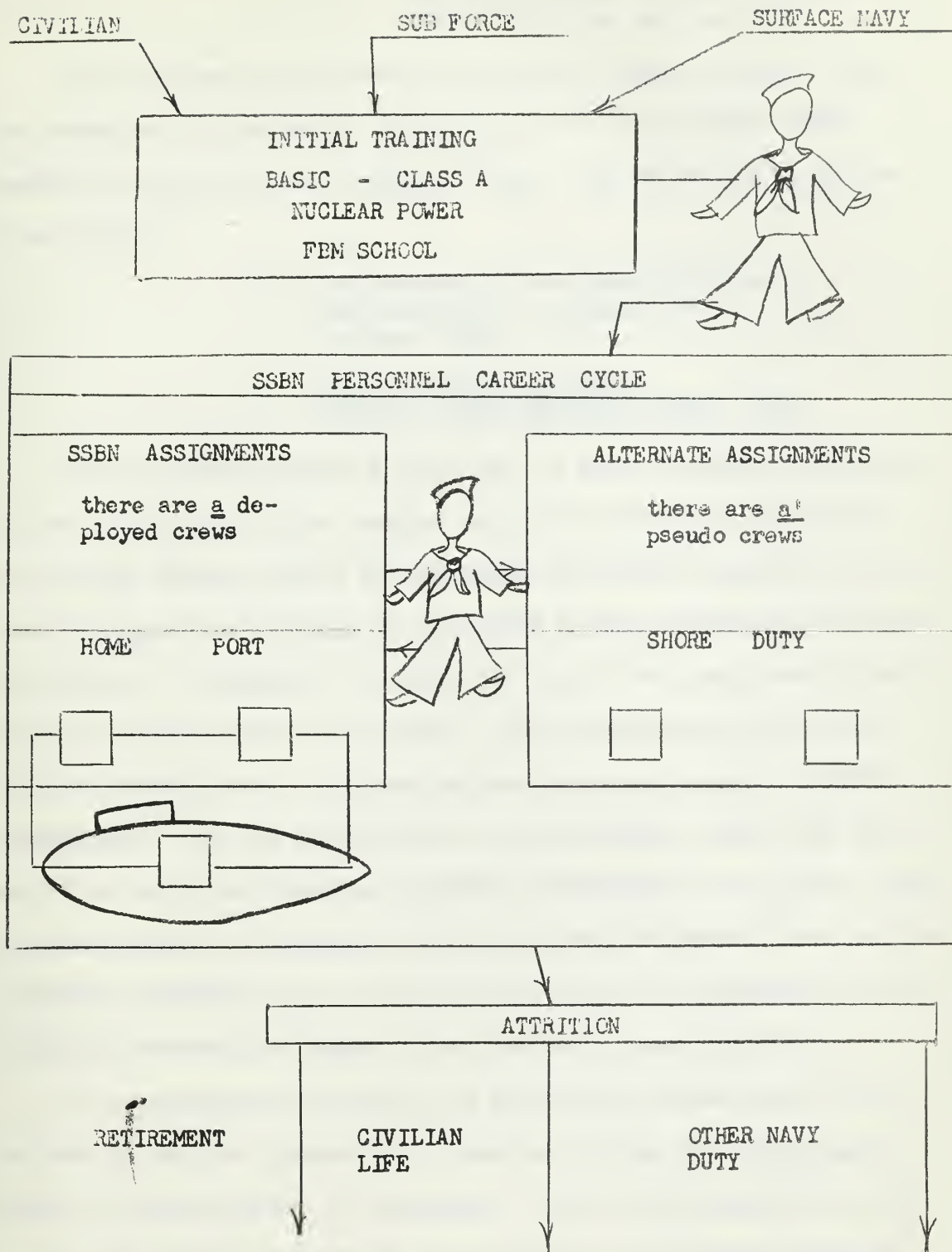


FIGURE 6. THE SSBN PERSONNEL SYSTEM

- (6) $A = a + a' =$ the total number of crews in the CAREER CYCLE per deployed boat

The fraction of his career that each man spends assigned to an FBM submarine will determine the number of men in the CAREER CYCLE needed for each billet in a deployed crew. Thus we are led to define T_c as follows:

- (7) $T_c =$ the fraction of the CAREER CYCLE an FBM submariner is assigned to a deployed SSBN

$$T_c = \frac{\text{Length of SSBN ASSIGNMENT}}{\text{Length of SSBN PERSONNEL CAREER CYCLE}}$$

The LONG RANGE OPERATION CYCLE and the SSBN PERSONNEL CYCLE need not be as carefully fitted together as the CREW PATROL CYCLE and the BOAT PATROL CYCLES, because the individual personnel assigned to a particular boat may be assigned or reassigned without interfering with the Boat cycles. For example, if a boat is to go to the yard, some of its personnel will be sent to other SSBN. These people are still in the SSBN ASSIGNMENT phase. The remaining are personnel going to ALTERNATE ASSIGNMENTS. When the boat returns to the Deployment Phase, the crews are filled with personnel from ALTERNATE ASSIGNMENTS or from other SSBN. Thus the movement of personnel between the SSBN ASSIGNMENT phase and the ALTERNATE ASSIGNMENT phase is not directly tied to the movement of the submarines between the phases of the LONG RANGE OPERATION CYCLE.

It is envisioned that until the FBM weapons system begins to be replaced by the next generation of deterrent forces that these basic cycles will characterize its operation. It is to be expected that the length of the cycles and the relative length of the phases within the cycles may change from time to time, or may be fitted together into various long range patterns. These considerations will be kept in mind throughout

the development of the model. We will not, however, anticipate a sudden change in the role of the FBM system in the defense of the nation. Thus, the operational commitments of each submarine may be planned to satisfactory precision for as long as ten years.

The operation has now been described in terms of four cycles. The fractions of each cycle spent in each phase have been defined and symbols have been defined to represent the number of crews in each cycle per deployed boat. In the next chapter, these concepts are used to formulate the desired mathematical analysis.

CHAPTER II

AN ANALYTICAL MODEL

In Chapter II we will develop the analytical method of comparing the various alternatives. First, we shall find the number of submarines needed to keep one on station. Next, the total input of personnel to the system over a period of years will be determined, and finally, the equations for the total costs over the same period of years for both the submarines and the personnel will be derived.

1. The Number of Boats Per Station.

We are interested in the number of submarines in the LONG RANGE OPERATIONAL CYCLE per boat on station. To do this, we first determine the number of deployed submarines per boat on station.

Part of each PATROL must be used in transit to and from station. Therefore, the fraction of the BOAT PATROL CYCLE spent on station, t_b , is related to t_s by a factor which shall be defined as

$$(8) \quad r = \frac{\text{Length of time on station}}{\text{Length of time on patrol; } T_0}$$

thus
$$r = t_s/t_b.$$

Then we can write

$$(9) \quad t_s = t_b \cdot r.$$

We shall call r the onstation factor.

The number of deployed boats per station is now easily computed.

Let

$$(10) \quad n_s = \text{the number of deployed boats per boat on station;}$$

then
$$n_s = \frac{1}{t_s}$$

Consideration of the fraction of the LONG RANGE OPERATIONAL CYCLE which a boat spends deployed leads to the number of submarines per station.

Let

$$(11) \quad N_S = \text{the number of submarines in the LONG RANGE OPERATIONAL CYCLE per boat on station;}$$

then

$$N_S = \frac{n_S}{T_b}$$

The number of submarines, N_S , leads to the determination of the number of stations which can be patrolled with a given number of boats. The values of n_S and N_S will provide inputs to the cost equations to be analyzed in section 3. However, before turning to the cost consideration, the personnel requirements over a period of time will be developed next.

2. The Long Range Personnel Requirements.

In this section, we will determine the number of personnel which will be required to maintain the personnel system at the desired level of training and numbers of crews over a long period of time.

The submarine is built and then can be expected to operate satisfactorily with the usual yard maintenance for a long period of time. On the other hand, personnel are subject to turnover. This turnover is a significant fraction of a crew each year. In addition to attrition, other members of the crew must be moved from billet to billet.

To provide and maintain the necessary crews to operate the submarine, it is necessary to first train the initial A crews per deployed boat and then replace the personnel lost due to attrition. To express the total input of personnel over a period of Y years in symbols,

Let

$$(12) \quad R = \begin{array}{l} \text{the fraction of the personnel in} \\ \text{the CAREER CYCLE lost by attrition} \\ \text{each year} \end{array}$$

and

$$I_b = \begin{array}{l} \text{total number of personnel input per} \\ \text{deployed submarine over } Y \text{ years} \\ \text{measured in crews;} \end{array}$$

then the desired result is

$$(13) \quad I_b = A + Y \cdot A \cdot R = A(1 + Y \cdot R)$$

To count personnel per station rather than per deployed boat, we simply multiply I_b by n_s , the number of deployed boats per station. Therefore the total personnel input per station is expressed as

$$I_s = \begin{array}{l} \text{the total number of personnel input per} \\ \text{boat on station for } Y \text{ years measured} \\ \text{in crews.} \end{array}$$

then

$$(14) \quad I_s = I_b \cdot n_s$$

3. The Cost Equations.

Although SSBN submarines are expensive, the crews cost money too. Of course, the crews are not purchased as equipment is but it does take money to recruit, train, pay and administrate the personnel required to man these SSBN crews. The term "crew cost" will be used to denote these expenses.

The total cost of the personnel over a long period of time consists of three factors. The first is the cost of training the initial input

of A crews per deployed boat.¹ The next factor is the cost of paying and feeding these crews.² Finally, we must include the cost of training the replacements which are required because of the attrition. Then for the total personnel cost to provide and maintain A crews per deployed boat over Y years, D_b , we must account for the initial training cost, the cost of replacing annual attrition, and the annual personnel expenses for pay, food, and administrative costs.

Let

(15) D_b = the total personnel cost per deployed boat over Y years

$$D_b = k \cdot S \cdot A + Y \cdot k \cdot S \cdot A \cdot R + Y \cdot S \cdot (A - h(A - a))$$

$$1 - h \left(\frac{Y \cdot S \cdot (A - h(A - a))}{Y \cdot S \cdot (A - h(A - a))} \right)$$

Similarly, the total personnel cost per boat on station over Y years

follows simply.

Let

(16) D_s = the total personnel cost per boat on station over Y years

$$D_s = D_b \cdot n_s$$

$$= \frac{42}{52} (A)$$

Ind. Train.

To put the cost equation in symbols, we define

(17) S = the annual cost of paying, feeding, transporting, and providing for the general welfare of the crew including the on-the-job and between-patrol training the crew receives,

$R \cdot S$ = the initial training cost of a crew

$h \cdot S$ = the amount saved per year by a crew in alternate assignment (pseudo crew);

¹Recall that $A = a + a'$ and defined in equation (6)

²The annual personnel costs will be modified to account for the fact that personnel in alternate assignments should not be included in the personnel costs per boat or station patrolled.

and recall that

- A is the total number crews deployed
and psuedo crews per deployed boat,
- a is the number of deployed crews
assigned per deployed boat,
- R is the fraction of the personnel of
the a crew lost to attrition each
year,
- n_s is the number of deployed boats per
boat on station.

We will now devise cost equations similar to equations (15) and (16) for the cost of the submarines themselves for a period of Y years. We begin by defining these symbols for the boat costs which do not include crew costs,

(18) K = the initial cost of the boat, missiles
and equipment,

M = the annual cost of maintaining and
operating a boat.

Included in the annual maintenance and operating cost is the cost of yard and tender repairs and other costs which are functions of the number of submarines which we have.

Thus the boat cost of one boat for Y years is

(19) $B = K + Y \cdot M.$

Because a boat will not be deployed at all times, we would like to know the boat cost in terms of its deployed time, hence we will define B_d , the boat cost per deployed boat over a period of Y years. Then

(20) $B_d = \frac{B}{T_b} \quad (T_b \leq 1)$

Similarly, for the boat costs per boat on station for a period of Y years we have

(21) $B_s = B \cdot N_s$

With the aid of these equations which are summarized in Table 1, we are equipped to do a cost comparison of alternative personnel policies.

4. The Choice and Effect of Upkeep Base Location.

The advanced base location enhances the fraction of time on station by decreasing the transit time. The use of the advanced base requires the crew to be away from home during both the PATROL and the UPKEEP period. One may consider using the home port for upkeep to allow the crews to be home more between patrols. The question which then arises is how much on station time is lost due to the longer transit and what is the resulting effect on the overall costs and personnel requirements.

In equation (9), we expressed the fraction of the BOAT PATROL CYCLE spent on station as

$$t_s = t_b \cdot r$$

where r is the on station factor. To evaluate t_s as a function of the variables, t_c and a , associated with the personnel policy, we will develop an expression for t_s as a function of a and t_c for a particular home port, advanced base and operation area.

First let

$$r = \begin{cases} r_a & \text{when the upkeep takes place in} \\ & \text{the advanced base} \\ r_h & \text{when the upkeep takes place in} \\ & \text{the home port,} \end{cases}$$

then write

$$t_s = \begin{cases} t_b \cdot r_a & \text{when the upkeep takes place in} \\ & \text{the advanced base} \\ t_b \cdot r_h & \text{when the upkeep takes place in} \\ & \text{the home port.} \end{cases}$$

Recall the discussion that led to equation (3a). When the upkeep takes place in an advanced base each crew is with the boat and ~~away~~ away from home, for an entire BOAT PATROL CYCLE. Then the division of the

BOAT PATROL CYCLE is not influenced by t_c or a .

Therefore when the upkeep takes place in an advanced base, t_b is independent of a and t_c . In this case the value of t_b may be chosen as large as possible consistent with the maintenance requirements of the boat. Let this value of t_b be called $t_{b_{max}}$. When the upkeep takes place in the home port, we can substitute $a \cdot t_c$ for t_b .

Thus

$$(22) \quad t_s = \begin{cases} t_{b_{max}} \cdot r_a & \text{if the upkeep takes place} \\ & \text{in an advanced base} \\ a \cdot t_c \cdot r_h & \text{if the upkeep takes place in} \\ & \text{the home port.} \end{cases}$$

A detailed analysis of the dependence of t_s on t_b , $t_{b_{max}}$, a , t_c , r_a , r_h and the choice between advanced base or home port upkeep is developed in the Appendix. Briefly the results are:

$$(23) \quad \begin{array}{ll} \text{if } a \cdot t_c = 1 & \text{then the upkeep should be in} \\ & \text{an advanced base;} \\ \text{if } a \cdot t_c \leq \frac{r_h}{r_a} & \text{then the upkeep should be in} \\ & \text{the home port;} \\ \text{if } \frac{r_h}{r_a} < a \cdot t_c < 1 & \text{then the upkeep locations} \\ & \text{should be a combination} \\ & \text{determined by the procedures} \\ & \text{in the Appendix:} \end{array}$$

and

$$(24) \quad t_s = \begin{cases} t_{b_{max}} \cdot r_a & \text{if the upkeep is to take} \\ & \text{place in an advanced base} \\ a \cdot t_c \cdot r_h & \text{if the upkeep is to take} \\ & \text{place in the home port.} \\ t_s & \text{if a combination of home} \\ & \text{port and advanced based} \\ & \text{upkeep locations is in-} \\ & \text{dicated. See the Appendix.} \end{cases}$$

Thus it is possible to determine for each value of a and t_c in a given geographical situation, which upkeep location should be used and the resulting value of t_s .

CHAPTER III

APPLICATION OF THE ANALYTICAL MODEL

In the first section of this chapter, the various alternative personnel policies are discussed, and the procedure for applying this model to their analysis is summarized. The second section offers four examples. In the third section, possible refinements to this model are discussed.

1. The Alternatives.

We now turn our attention to the alternatives which this thesis proposes to compare using the analytic model of Chapter II.

To simplify the procedure of comparing personnel policies and the resulting operational scheme, it will be convenient to list precisely the characteristics of an SSBN personnel policy. The symbols for these characteristics have been defined in Chapters I and II. Therefore a personnel policy is characterized by:

- (a) The number of deployed crews per deployed boat, a .
- (b) The career rotation in terms of years of SSBN sea duty for each year of shore duty or other sea duty. The PERSONNEL CAREER CYCLE characterizes this rotation.
- (c) The fraction of the CREW PATROL CYCLE that each crew is away from home, t_c .

It must be recognized that such a policy will not be devised without consideration of the operation of the submarine. In fact, it is the interrelation between the personnel system and the submarine system operational that we seek to study in order to find the operational scheme which is optimized with respect to both personnel and equipment.

As a result of specifying the personnel policy as indicated above, certain of the characteristics of the boat operation are determined.

For instance, the fraction of the BOAT PATROL CYCLE spent on station, t_s , is determined from equations (23) and (24), while others must be specified because they are independent of the PERSONNEL POLICY.

The cost parameters which are inputs to the analysis must be determined from budgetary data and personnel statistics. The attrition rate is a function of the personnel policy because an attractive career will retain more of the personnel. However, we will be able to estimate only qualitatively the attrition rate because of the unpredictability of the personnel reaction to an alternative personnel policy.

Here is a step by step procedure for determining the personnel requirements, boat requirements, and associated costs of an alternative PERSONNEL POLICY:

- (a) Specify the PERSONNEL POLICY by specifying A , a , and t_c .
- (b) Specify the independent boat parameters, T_b and $t_{b_{max}}$, and N (if applicable.)
- (c) Determine the location of the upkeep bases with respect to the operation area in order to specify r_a and r_h .
- (d) Determine a feasible operation plan that has the characteristics of (a), (b), and (c) above.
- (e) Determine the use of the upkeep locations from equation (23).
- (f) Determine the use of t_s from equation (24)
- (g) Determine n_s and N_s from equations (10) and (11).
- (h) Determine S , S_k , S_h , M , and K from budgetary and personnel data. Specify Y , the number of years considered.
- (i) Estimate R or express the remaining equations as a function of R .
- (j) Determine I_b and I_s from equations (13) and (14).
- (k) Determine D_b and D_s from equations (15) and (16).
- (l) Determine B , B_d and B_s from equations (19), (20), and (21)

Table 1 provides a summary of equations which are to be used in the analysis of an alternative personnel policy and suggests a format which may be helpful.

In the development of the analysis, we have expressed the results in two ways, as the costs or numbers of personnel required per boat on station or per deployed boat. Other measures of effectiveness which are appropriate are total cost and capability of a fixed number of boats, boat cost, personnel costs, personnel required, and number of stations which can be patrolled by a fixed number of boats. For this purpose, let N^* be the number of boats in the LONG RANGE OPERATIONAL CYCLE. The equations are easily derived from the variables already defined and they are shown in table 2.

Equations (10) and (11) give the number of submarines for two situations. It will be illustrative to observe the analogous factors for the numbers of crews.

For this let

c_s = the number of deployed crews per
boat on station,

C_s = the number of crews in the CAREER CYCLE
per boat on station,

then we just multiply A and a by the number of deployed boats per station to get

$$(25) \quad c_s = a \cdot n_s$$

$$(26) \quad C_s = A \cdot n_s$$

Table 2 gives a summary of the suggested measures of effectiveness, which can be used to place the results of this analysis in perspective.

TABLE 1

EQUATIONS AND FORMAT FOR THE ANALYSIS

Specify the Personnel Policy with

$$(a) \quad A \quad \text{-----} \quad a \quad \text{-----} \quad t_c \quad \text{-----}$$

Specify the independent boat parameters

$$(b) \quad T_b \quad \text{-----} \quad t_{b_{\max}} \quad \text{-----} \quad N^* \quad \text{-----}$$

Determine the effect of the distances to the stations

$$(c) \quad r_a \quad \text{-----} \quad r_h \quad \text{-----}$$

(d) Work out operational complications if any. Ensure that the cycle lengths are feasible.

(e) Determine which upkeep base is to be used

if $a \cdot t_c = 1$ then use the advanced base for upkeep

$a \cdot t_c \leq \frac{r_h}{r_a}$ then use the home port for upkeep

$\frac{r_h}{r_a} < a \cdot t_c < 1$ then use a combination of home port and advanced base for upkeep in accordance with the Appendix

(f) Determine the fraction of the BOAT PATROL CYCLE spent on station

$$t_s = \begin{cases} t_{b_{\max}} \cdot r_a & \text{the advanced base is used for upkeep} \\ a \cdot t_c \cdot r_h & \text{when the home port is used for upkeep} \\ t_s & \text{when a combination of home port and advanced base is used for upkeep in accordance with the Appendix} \end{cases}$$

(g) Determine the number of deployed boats per boat on station

$$n_g = \frac{1}{t_s}$$

Determine the number of boats in the LONG RANGE OP-CYCLE per boat on station

$$N_s = \frac{n_g}{T_b}$$

TABLE 1 continued

- (h) Determine the number of deployed crews per boat on station

$$c_s = a \cdot n_s$$

Determine the number of crews in the PERSONNEL CAREER CYCLE per boat on station

$$C_s = A \cdot n_s$$

Determine from data of conjecture values of the cost parameters

$$\begin{array}{ccc} S & \underline{\hspace{2cm}} & k \underline{\hspace{2cm}} \quad h \underline{\hspace{2cm}} \\ K & \underline{\hspace{2cm}} & M \underline{\hspace{2cm}} \quad Y \underline{\hspace{2cm}} \end{array}$$

- (i) Estimate the attrition rate R.
 (j) Determine the total personnel input per deployed boat for Y years

$$I_b = A(1 + Y \cdot R)$$

Determine the total personnel input per boat on station for Y years

$$I_s = n_s \cdot I_b$$

- (k) Determine the total personnel costs per deployed boat for Y years

$$D_b = S \cdot k \cdot I_b + Y \cdot S \cdot A + Y(A - a) \cdot h \cdot S \quad y, a, h, S$$

Determine the total personnel costs per boat on station for Y years

$$D_s = n_s D_b$$

- (l) Determine the boat costs for Y years

$$B = K + Y M$$

Determine the boat cost per deployed boat for Y years

$$B_d = \frac{B}{T_b}$$

Determine the boat cost per boat on station for Y years

$$B_s = B N_s$$

TABLE 2

SUGGESTED MEASURES OF EFFECTIVENESS

1. Total requirements for 1 Deployed Boat

$\frac{1}{T_b}$ = Total number of submarines in the
LONG RANGE OPERATIONAL CYCLE

I_b = Total number of personnel input
(measured in crews)

B_b = Total cost of the submarines

D_b = Total cost of the personnel

2. Total requirements for 1 station patrolled continuously

N_s = Total number of submarines in the
LONG RANGE OPERATIONAL CYCLE

I_s = Total number of personnel (in crews)

B_s = Total boat cost

D_s = Total crew costs

3. Capability of a total of N^* boats in the LONG RANGE OPERATIONAL CYCLE

$N^* \cdot T_b$ = the number of boats deployed

$\frac{N^*}{N_s}$ = the number of stations patrolled

4. Total cost of N^* boats over a period of Y years

$N^* \cdot 2BM$ = Total Boat Costs

$N^* \cdot T_b D_b$ = Total Personnel Costs

$N^* \cdot T_b I_b$ = Total Personnel Input

2. Examples

Alternative 1: Alternative 1 is approximately the present plan.

- (a) $A = 3$ $a = 2$ $t_c = \frac{1}{2}$
- (b) $T_b = 5/6$ $t_{b_{\max}} = 2/3$ $N^* = 41$
- (c) $r_a = 1$ $h = .78$
- (d) The present Op plan.
- (e) Upkeep in advanced base.
- (f) $t_s = 2/3$
- (g) $n_s = 1.5$ $N_s = 1.8$
- (h) $c_s = 3$ $C_s = 4.5$
- (i) $R = .3$
- (j) $k = 3$ $h = 1$ $Y = 20$ S, M, K unspecified
- (k) $I_b = 3(1 - 20R) = 21$ $I_s = 31.5$
- (l) $D_b = 100S$
 $D_s = 150.S$
- (m) $B = K - 20M$ $B_d = 1.2B$
 $B_s = 1.8B$

With 41 boats, we get 22.8 stations patrolled at a cost of 150.3 ⁷¹ 1.8B per station B. 1.8B

Alternative 2. If we operate with one deployed crew without encreasing the "workload" of the crew -- we have this situation:

- (a) $A = 1.5$ $a = 1$ $t_c = \frac{1}{2}$
- (b) $T_b = 5/6$ $t_{b_{\max}} = 2/3$ $N^* = 41$ $Y = 20$ years
- (c) $r_a = 1$ $h = .78$
- (d) The present plan modified as follows
- (e) Upkeep in the home port
- (f) $t_s = .39$
- (g) $n_s = 2.56$ $N_s = 3.08$

$$(h) \quad c_s = 2.56 \quad C_s = 3.84$$

$$(i) \quad R = .3$$

$$(j) \quad k = 3 \quad h = 1 \quad S, M \text{ unspecified.}$$

$$(k) \quad I_b = 1.5 (1 \ 7) = 10.5 \quad I_s = 20.69$$

$$(l) \quad D_b = 5.31.5 \quad 35S - 10S = 56.55$$

$$D_s = 133S$$

$$(m) \quad B = K \quad 20M, B_d = 1.2B, B_s = 3.08B$$

41 boats will patrol 13.3 stations. It will take 70 boats to patrol 22.3 stations at a cost of 133S 3.08B per station

Alternatives 3(A) and 3(B)

$$(a) \quad A = 1.5 \quad a = 1 \quad t_c = 2/3$$

$$(b) \quad T_b = 5/6 \quad t_{b_{\max}} = 2/3 \quad N^* = 41$$

$$(c) \quad r_a = 1 \quad n = .78$$

(d) no problems.

(e) upkeep at homeport

$$(f) \quad t_s = .515$$

$$(g) \quad n_s = 1.94 \quad N_s = 2.33$$

$$(h) \quad c_s = 1.94 \quad C_s = 2.92$$

$$(i) \quad S, M \text{ unspecified, } k = 3, \quad h = 1$$

3(A):

(j) $R = .3$

(k) $I_b = 10.5$

$I_s = 20.4$

(l) $D_b = 56.5 \cdot S$

$D_s = 110 \cdot S$

(m) $B = K \quad 20M$

$B_d = 1.2B$

$B_s = 2.33B$

In this case we have saved on both crews and boats over alternate 2 but we have neglected a possible increase in R

3(B):

(j) $R = .4$

(k) $I_b = 13.5$

$I_s = 26.2$

(l) $D_b = 65.5 \cdot S$

$D_s = 127 \cdot S$

(m) $B = K \quad 20M$

$B_d = 1.2B$

$B_s = 2.33B$

If the increase in R is .1 over at 3(A) then this case gives the result this case is the best of the examples, if R is realistic.

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

1. Conclusions.

The model presented here is capable of performing an analysis of the CSBN operation to determine long range personnel requirements and associated costs for the purpose of comparing various personnel policies.

An analysis can be made of the entire system or of a portion of the system. Thus a particular part of the system such as the home port and advanced base which support a particular operation area can be studied, taking into consideration only a portion of the crew.

The results of the study of these costs and requirements are influenced to a considerable degree by the actual value of the personnel attrition rate. This factor will prove to be the most difficult to determine accurately because it depends in part on the crews' reaction to the personnel policy. Expressing the results as functions of the attrition rate will lead to a useful sensitivity analysis.

The model developed in this paper will provide a basis for further work on this problem as suggested below in the recommendations.

2. Recommendations.

Further application of this model will require these steps to be taken:

- (a) Refine the model to take into account the variations between the groups of personnel which have different characteristics.
- (b) Refine the model to take into account the various home port-advanced base-operation area complexes.
- (c) Refine the technique of performing a sensitivity analysis of the attrition rates of the various personnel groups.

- (d) Program the sensitivity analysis for a digital computer to handle the computations required by (c) above.
- (e) Refine the cost equations to reflect the effect of cost discounting over time. This step is considered quite important in view of the long time increment (20 years) used to characterize the life of each of the submarines and length of the careers of the personnel.
- (f) Pursual of step (d) leads one to suggest that the various parameters should be made functions of time to consider not only cost discounting but other time dependent aspects such as a transition from one mode of operation to another.
- (g) Finally, it shall be necessary to devise and implement a data collection plan. This plan will have two phases. The first will be an investigation of the types of information available and its location. The second will be a plan to collect and reduce, to the required format, the necessary data to perform an analysis.

Although a considerable amount of work to improve this model has been indicated, the comparisons produced by the model in its present form do provide some insight into the implications of the employment of various numbers of crews for each boat and establishing various career patterns.

APPENDIX

The purpose of this Appendix will be to verify equations (23) and (24) and to show that

$$\hat{t}_s = \frac{a \cdot t_c - \hat{p}}{(1-\hat{p})} \left[p(r_a - r_h) + r_h \right] \quad \text{if } \frac{r_h}{r_a} < a \cdot t_c < 1$$

where \hat{p} = the optimum fraction of the UPKEEPS of a BOAT DEPLOYMENT phase which are to take place in the advanced base;

$$\hat{p} = 1 - \sqrt{1 - \frac{a \cdot t_c \cdot r_a - r_h}{(r_a - r_h)}}.$$

Equation (23) of Chapter II states

if $a \cdot t_c = 1$

then the upkeep should be in the advanced base.

if $a \cdot t_c \leq \frac{r_h}{r_a}$

then the upkeep should be in the home port.

if $\frac{r_h}{r_a} < a \cdot t_c < 1$

then the upkeep locations should be a combination determined by the procedures of Appendix.

Equation (24) of Chapter II states

$$t_s = \begin{cases} t_{cmax} \cdot r_a \\ a \cdot t_c \cdot r_h \\ \hat{t}_s \end{cases}$$

if the upkeep is to take place in the advanced base.

if the upkeep is to take place in the home port.

if a combination of home port and advanced base upkeep location is indicated. See Appendix.

First we shall define a variable, \bar{t}_s , which is similar to t_s of equation (9). This new variable will be a function of $a \cdot t_c, r_a, r_h$ and a variable, p , which reflects the amount of use of the advanced base.

To shorten the notation for the lengths of some of the operational

phases, let

$$\begin{aligned}
 \text{(A1)} \quad \text{LOP} &= \text{Length of the PATROL} \\
 \text{LPC} &= \text{Length of the BOAT PATROL CYCLE} \\
 \text{LCTa} &= \text{Length of the transit to and from the} \\
 &\quad \text{advanced base.} \\
 \text{LOTh} &= \text{Length of the transit to and from the} \\
 &\quad \text{home port.}
 \end{aligned}$$

Suppose over a BOAT DEPLOYMENT, which consists of a number P of BOAT PATROL CYCLES, there are to be P* UPKEEP periods which take place in an advanced base and (P-P*) UPKEEP periods which take place in the home port. With this notion it will be convenient to take a longer range look at the time fractions.

let

$$\begin{aligned}
 \bar{t}_a &= \frac{\left\{ \begin{array}{l} \text{length of time on station during a BOAT} \\ \text{DEPLOYMENT} \end{array} \right\}}{\text{(length of the BOAT DEPLOYMENT)}} \\
 \text{(A2)}
 \end{aligned}$$

$$\bar{t}_a = \frac{P*(\text{LOP}-\text{LCTa}) + (P-P*) \cdot (\text{LOP}-\text{LOTh})}{P(\text{LPC})}$$

now let

p = the fraction of UPKEEP periods of a BOAT DEPLOYMENT phase which take place in the advanced base;

$$\begin{aligned}
 \text{(A3)} \quad p &= \frac{P^*}{P}; \\
 0 &\leq p \leq 1.
 \end{aligned}$$

From equation (9), notice that

$r_a \cdot t_b$ = the fraction of BOAT PATROL CYCLE spent on station when the advanced base is used for upkeep,

then

$$r_a \cdot t_b = \frac{\text{LOP}-\text{LCTa}}{\text{LPC}}.$$

Likewise

$r_h \cdot t_c$ = the fraction of the BOAT PATROL CYCLE spent on station when the home port is used for upkeep.

Then

$$r_h \cdot t_c = \frac{LOF - LOTh}{LPC}$$

therefore

$$(A4) \quad \bar{t}_c = p(t_b \cdot r_a) + (1-p)(t_b \cdot r_h)$$

Notice that $p = 1$ implies that all UPKEEP is to take place in an advanced base;

$p = 0$ implies that all UPKEEP is to take place in the home port;

$0 < p < 1$ implies that there will be a plan to use both home port and advanced bases for UPKEEP.

We now seek to eliminate t_b from our expression for \bar{t}_c .

Begin by defining \bar{t}_c as a function of a , t_b , and p .

Let

(A6) \bar{t}_c = the fraction of time of a BOAT DEPLOYMENT that each deployed crew spends away from home;

$$\bar{t}_c = \frac{\frac{F(LOP)}{a} + \frac{F^*}{a} (\text{length of UPKEEP})}{F \cdot LPC};$$

$$\bar{t}_c = \frac{t_b}{a} + \frac{p(LPC - LOP)}{a(LPC)}$$

$$= \frac{t_b}{a} + \frac{p(1-t_b)}{a}$$

Finally we put \bar{t}_c in the form of equation (3), which is

$$(A7) \quad \bar{t}_c = \begin{cases} \frac{t_b + p(1-t_b)}{a} & \text{if } p < 1 \\ \frac{1}{a} & \text{if } p = 1 \end{cases}$$

Now rewrite (A7) for t_b as a function of $a \cdot t_c$ and p .

The result is

$$(A8) \quad t_b = \begin{cases} \frac{a \cdot t_c - p}{1-p} & \text{if } p < 1 \\ \text{independent of } a \cdot t_c & \text{if } p = 1 \end{cases}$$

Recall the discussion which defined $t_{b_{\max}}$ in section II-4.

With this in mind, we write

$$(A9) \quad t_b = \begin{cases} \frac{a \cdot t_c - p}{1-p} & \text{if } p < 1 \\ t_{b_{\max}} & \text{if } p = 1 \end{cases}$$

and require that

$$(A10) \quad t_b \leq t_{b_{\max}} \leq 1$$

By combining equations (A7) and (A12) we arrive at the desired for the value of t_s , which is now written as \bar{t}_s , to indicate that it is a function of p as well as $a \cdot t_c, r_a$, and r_h . Thus we have

$$(A11) \quad \bar{t}_s = \begin{cases} t_{b_{\max}} \cdot r_a & \text{if } p = 1 \\ \frac{a \cdot t_c - p}{1-p} \cdot [p(r_a - r_h) + r_h] & \text{if } 0 \leq p < 1 \end{cases}$$

We now prepare to determine the optimum value of \bar{t}_s which we will call t_s for each value of $a \cdot t_c$, r_h , and r_a .

From equations (A7) and (A10) it can be shown that

$$(A12) \quad a \cdot t_c = 1 \quad \text{if and only if} \quad p = 1.$$

As a result

$$a \cdot t_c = 1 \quad \text{implies that} \quad \bar{t}_s = t_{b_{\max}} \cdot r_a = \hat{t}_s$$

Equations (A3) and (A8) impose this constraint

$$(A13) \quad 0 \leq p \leq a \cdot t_c$$

Assuming the value of p is independent of cost consideration, we shall maximize \bar{t}_s over values of p . The procedure will be to differentiate \bar{t}_s and set the result equal to zero, then solve for p subject to (A13). Let the solution be called \hat{p} .

$$(A14) \quad \frac{d\bar{t}_s}{dp} = \frac{(r_a - r_h)p^2 - 2(r_a - r_h)p + a \cdot t_c \cdot r_a - r_h}{(1-p)^2};$$

$$\hat{p} = \frac{2(r_a - r_h) \pm \sqrt{4(r_a - r_h)^2 - 4(r_a - r_h)a \cdot t_c \cdot r_a - r_h}}{2(r_a - r_h)}$$

$$\hat{p} = 1 \pm \sqrt{1 - \frac{a \cdot t_c \cdot r_a - r_h}{r_a - r_h}};$$

but $0 \leq p \leq 1$

therefore

$$\hat{p} = 1 - \sqrt{1 - \frac{a \cdot t_c \cdot r_a - r_h}{r_a - r_h}} \quad \text{for } \frac{r_h}{r_a} < a \cdot t_c < 1$$

and let $\hat{p} = 0$ for $a \cdot t_c \leq \frac{r_h}{r_a}$

We can now write

$$(A15) \quad \hat{p} = 1 \quad \text{if} \quad a \cdot t_c = 1;$$

$$\hat{p} = 0 \quad \text{if} \quad a \cdot t_c \leq \frac{r_h}{r_a};$$

$$\hat{p} = 1 - \sqrt{1 - \frac{a \cdot t_c \cdot r_a - r_h}{r_a - r_h}} \quad \text{if } \frac{r_h}{r_a} < a \cdot t_c < 1$$

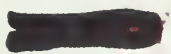
and finally

$$(A16) \quad t_s = \begin{cases} t_{\max} \cdot r_a & \text{if } a \cdot t_c = 1 \\ a \cdot t_c \cdot r_h & \text{if } a \cdot t_c \leq \frac{r_h}{r_a} \\ \frac{a \cdot t_c - \hat{p}}{(1-\hat{p})} \cdot [p(r_a - r_h) + r_h] & \text{if } \frac{r_h}{r_a} < a \cdot t_c < 1 \end{cases}$$

LIST OF SYMBOLS

- A - The total number of crews in the CAREER CYCLE per deployed boat.
- a - The number of deployed crews per deployed boat.
- a' - The number of pseudo crews per deployed boat.
- B - The boat cost of one boat for Y years.
- B_d - The boat cost per deployed boat for Y years.
- B_s - The boat cost per boat on station for Y years.
- C_s - The number of crews in the CAREER CYCLE per boat on station.
- c_s - The number of deployed crews per boat on station.
- D_b - The total personnel cost per deployed boat over Y years.
- D_s - The total personnel cost per boat on station over Y years.
- I_b - The total number of personnel input per deployed boat over Y years measured in crews.
- I_s - The total number of personnel input per boat on station for Y years measured in crews.
- K - The initial cost of an FEM submarine.
- M - The annual cost of maintaining and operating a boat.
- N* - The number of boats in the LONG RANGE OPERATIONAL CYCLE.
- N_s - The number of boats in the LONG RANGE OPERATIONAL CYCLE per boat on station.
- n_s - The number of deployed boats per boat on station.
- R - The fraction of the personnel in the CAREER CYCLE lost by attrition each year.
- r - The fraction of the PATROL spent on station. The on station factor.
- r_a - The on station factor for the advanced base.
- r_h - The on station factor for the home port.
- S - The annual cost of paying, feeding, transporting and providing for the general welfare of the crew including the on the job training and between patrol training the crew receives.

- S.h - The amount saved per year per crew in the alternate assignment.
- S.k - The initial training cost of a crew.
- T_D - The fraction of the LONG RANGE OPERATIONAL CYCLE spent in the DEPLOYMENT phase.
- t_D - The fraction of the BOAT PATROL CYCLE spent on PATROL.
- T_C - The fraction of the CAREER CYCLE an FBM submarineer is assigned to a deployed boat.
- t_C - The fraction of the CREW PATROL CYCLE that the crew is away from home.
- t_S - The fraction of the BOAT PATROL CYCLE spent on station.
- Y - The number of years to be considered for the long range costs and requirements.



thesB177

DUDLEY KNOX LIBRARY



3 2768 00407059 9

DUDLEY KNOX LIBRARY